

## Octonion Variance Sieve on the Associator

When the associator

$$\frac{1}{2} [a*(b*c) - (a*b)*c]$$

is cranked through the Octonion Variance Sieve Process<sup>[1]</sup>, the algebraic invariant portion of the result is identically zero. This does not imply the algebraic invariant portion of any Octonion product string is fully associative.

Take for instance the algebraic invariant unit product

$$u_3 [(u_5 u_3) (u_1 u_5)]$$

This is a unit product string that involves the following 4 unit multiplication permutations

$$\begin{aligned} &(u_1 u_5 u_4) \\ &(u_5 u_3 u_6) \\ &(u_6 u_4 u_2) \\ &(u_1 u_2 u_3) \end{aligned}$$

These are the 4 permutations that do not include the unit 7. When any valid<sup>[1]</sup> Octonion algebra change is made, either all, none, or two of these permutations will be negated. This leaves the resultant sign unchanged for any possible algebra.

If associative modification to the product order is attempted, say right to left, the 4 permutations involved are now

$$\begin{aligned} &(u_1 u_5 u_4) \\ &(u_3 u_7 u_4) \\ &(u_5 u_7 u_2) \\ &(u_1 u_2 u_3) \end{aligned}$$

These are the 4 permutations that do not include the unit 6, not the same grouping. The result is the same unit, but the sign changes. Associativity is broken, even though the result is again an invariant form.

The variant results of the Octonion Variance Sieve Process are given below in the following minimum distances.

For minimum distance 1:  $\frac{1}{2}[\text{SL}(123)+\text{SR}(321)]$

$$\begin{aligned} [4] &-a5b6c7+a5b7c6+a6b5c7-a6b7c5-a7b5c6+a7b6c5 \\ [5] &+a4b6c7-a4b7c6-a6b4c7+a6b7c4+a7b4c6-a7b6c4 \\ [6] &-a4b5c7+a4b7c5+a5b4c7-a5b7c4-a7b4c5+a7b5c4 \\ [7] &+a4b5c6-a4b6c5-a5b4c6+a5b6c4+a6b4c5-a6b5c4 \end{aligned}$$

For minimum distance 1:  $\frac{1}{2}[\text{SL}(123)-\text{SR}(321)]$

## Octonion Variance Sieve on the Associator

Null

For minimum distance 2:  $\frac{1}{2}[\text{SL}(761)+\text{SR}(167)]$

[ 2 ]+a3b4c5-a3b5c4-a4b3c5+a4b5c3+a5b3c4-a5b4c3  
[ 3 ]-a2b4c5+a2b5c4+a4b2c5-a4b5c2-a5b2c4+a5b4c2  
[ 4 ]+a2b3c5-a2b5c3-a3b2c5+a3b5c2+a5b2c3-a5b3c2  
[ 5 ]-a2b3c4+a2b4c3+a3b2c4-a3b4c2-a4b2c3+a4b3c2

For minimum distance 2:  $\frac{1}{2}[\text{SL}(761)-\text{SR}(167)]$

Null

For minimum distance 3:  $\frac{1}{2}[\text{SL}(572)+\text{SR}(275)]$

[ 1 ]-a3b4c6+a3b6c4+a4b3c6-a4b6c3-a6b3c4+a6b4c3  
[ 3 ]+a1b4c6-a1b6c4-a4b1c6+a4b6c1+a6b1c4-a6b4c1  
[ 4 ]-a1b3c6+a1b6c3+a3b1c6-a3b6c1-a6b1c3+a6b3c1  
[ 6 ]+a1b3c4-a1b4c3-a3b1c4+a3b4c1+a4b1c3-a4b3c1

For minimum distance 3:  $\frac{1}{2}[\text{SL}(572)-\text{SR}(275)]$

Null

For minimum distance 4:  $\frac{1}{2}[\text{SL}(653)+\text{SR}(356)]$

[ 1 ]+a2b4c7-a2b7c4-a4b2c7+a4b7c2+a7b2c4-a7b4c2  
[ 2 ]-a1b4c7+a1b7c4+a4b1c7-a4b7c1-a7b1c4+a7b4c1  
[ 4 ]+a1b2c7-a1b7c2-a2b1c7+a2b7c1+a7b1c2-a7b2c1  
[ 7 ]-a1b2c4+a1b4c2+a2b1c4-a2b4c1-a4b1c2+a4b2c1

For minimum distance 4:  $\frac{1}{2}[\text{SL}(653)-\text{SR}(356)]$

Null

For minimum distance 5:  $\frac{1}{2}[\text{SL}(145)+\text{SR}(541)]$

[ 2 ]-a3b6c7+a3b7c6+a6b3c7-a6b7c3-a7b3c6+a7b6c3  
[ 3 ]+a2b6c7-a2b7c6-a6b2c7+a6b7c2+a7b2c6-a7b6c2  
[ 6 ]-a2b3c7+a2b7c3+a3b2c7-a3b7c2-a7b2c3+a7b3c2  
[ 7 ]+a2b3c6-a2b6c3-a3b2c6+a3b6c2+a6b2c3-a6b3c2

For minimum distance 5:  $\frac{1}{2}[\text{SL}(145)-\text{SR}(541)]$

Null

For minimum distance 6:  $\frac{1}{2}[\text{SL}(246)+\text{SR}(642)]$

[ 1 ]-a3b5c7+a3b7c5+a5b3c7-a5b7c3-a7b3c5+a7b5c3  
[ 3 ]+a1b5c7-a1b7c5-a5b1c7+a5b7c1+a7b1c5-a7b5c1  
[ 5 ]-a1b3c7+a1b7c3+a3b1c7-a3b7c1-a7b1c3+a7b3c1  
[ 7 ]+a1b3c5-a1b5c3-a3b1c5+a3b5c1+a5b1c3-a5b3c1

For minimum distance 6:  $\frac{1}{2}[\text{SL}(246)-\text{SR}(642)]$

## Octonion Variance Sieve on the Associator

Null

For minimum distance 7:  $\frac{1}{2}[\text{SL}(347)+\text{SR}(743)]$

[1]  $-a_2b_5c_6+a_2b_6c_5+a_5b_2c_6-a_5b_6c_2-a_6b_2c_5+a_6b_5c_2$   
[2]  $+a_1b_5c_6-a_1b_6c_5-a_5b_1c_6+a_5b_6c_1+a_6b_1c_5-a_6b_5c_1$   
[5]  $-a_1b_2c_6+a_1b_6c_2+a_2b_1c_6-a_2b_6c_1-a_6b_1c_2+a_6b_2c_1$   
[6]  $+a_1b_2c_5-a_1b_5c_2-a_2b_1c_5+a_2b_5c_1+a_5b_1c_2-a_5b_2c_1$

For minimum distance 7:  $\frac{1}{2}[\text{SL}(347)-\text{SR}(743)]$

Null

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June 28, 2008

### Bibliography

<sup>[1]</sup> © 2008 Richard Lockyer

[http://www.octospace.com/files/Octonion Algebra and its Connection to Physics.pdf](http://www.octospace.com/files/Octonion_Algebra_and_its_Connection_to_Physics.pdf)